Simplified Expressions for the Calculation of the Contribution of the Heavy Components to the Transport Coefficients of Partially Ionized Gases

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Simplified expressions for the calculation of the contribution of the heavy components to the viscosity and to the translational thermal conductivity have been tested for pure and mixed plasmas.

Introduction

It is known[1] that the contribution of the heavy particles to the viscosity and to the translational thermal conductivity of a partially ionized gas can be calculated independently of the corresponding contribution of the electrons, according to

$$\eta = \eta_{\mathrm{H}} + \eta_{\mathrm{E}} \cong \eta_{\mathrm{H}}; \quad \lambda = \lambda_{\mathrm{H}} + \lambda_{\mathrm{E}} . \tag{1}$$

Moreover, since the slow convergence of the Chapman-Enskog method for the calculation of transport coefficients is mostly due to the electrons, η_{H} and λ_{H} can be calculated by means of the lower approximations of the Chapman-Enskog method[2].

These expressions appear, however, to be too sophisticated, in particular if one considers the accuracy of the input data (i. e. cross sections, composition[3]).

On the other hand, the approximations commonly used for neutral gases (i. e. the Buddenberg-Wilke expression for the viscosity and the Mason-Saxena equation for the translational thermal conductivity) are known to gice very poor results, when applied to partially ionized gases[4, 5].

The aim of this work is to investigate the possibility of using for the calculation of η_{H} and λ_{H} the simplified expressions recently tested by LEE and BOBBIT[6] for dissociating gases and to show the causes of the slow convergence of these expressions in the range of partial ionization. Atmospheric Helium (He, He^+, e), Argon (Ar, Ar^+, Ar^{++}, e), Xenon (Xe, Xe^+, Xe^{++}, e) and Nitrogen (N_2, N, N^+, N^{++}, N^{+++}, e) plasmas will be taken as examples. Results for mixtures of rare gases with nitrogen are also reported. The collision integrals of the relevant interactions as well as the composition of the plasmas under consideration have been taken from previous works[7, 8].

Transport Coefficients

a) Viscosity

According to HIRSCHFELDER et al. the first approximation of the Chapman-Enskog method for the calculation of the viscosity of a n-component mixture may be written in the form[9]

$$\eta_1 = - \begin{vmatrix} H_{11} & \cdots & H_{1n} & x_1 \\ \vdots & & \vdots & \\ H_{n1} & \cdots & H_{nn} & x_n \\ x_1 & & x_n & 0 \end{vmatrix} \Bigg/ \begin{vmatrix} H_{11} & \cdots & H_{1n} \\ \vdots & & \vdots \\ H_{n1} & \cdots & H_{nn} \end{vmatrix} , \tag{2}$$

$$H_{ii} = \frac{x_i^2}{[\eta_i]_1} + \sum_{\substack{k=1 \\ k \neq i}}^{n} \frac{2\, x_i\, x_k}{(M_i + M_k)} \frac{R\,T}{p[\mathcal{D}_{ik}]_1} \left[1 + \frac{3}{5} \frac{M_k}{M_i} A^*_{ik} \right] \tag{2 a}$$

$$H_{ij} = - \frac{2\, x_i\, x_j}{(M_i + M_j)} \frac{R\,T}{p[\mathcal{D}_{ij}]_1} \left[1 - \tfrac{3}{5} A^*_{ij} \right] , \tag{2 b}$$

$$A^*_{ij} = \Lambda^{*\,(2,2)}_{ij} / \Lambda^{*\,(1,1)}_{ij} . \tag{2 c}$$

Equation (2) can be expanded as

$$\eta_1 = \sum_{i=1}^{n} \frac{x_i^2}{H_{ii}} - \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{u} \frac{x_i\, x_j\, H_{ij}}{H_{ii}\, H_{jj}} + \dots . \tag{3}$$

The first term which can be derived from Eq. (2) by assuming all $H_{ij} = 0$ (i. e. the A^*_{ij}'s $= \frac{5}{3}$) will be referred to as the first approximation[10] and the first two terms to as the second approximation.

Thus

$$\eta_{\mathrm{f}} = \sum_{i=1}^{n} x_i^2 / H_{ii} , \tag{4}$$

$$\eta_{\mathrm{s}} = \sum_{i=1}^{n} x_i^2 / H_{ii} - \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} (x_i\, x_j\, H_{ij}) / H_{ii}\, H_{jj} . \tag{5}$$

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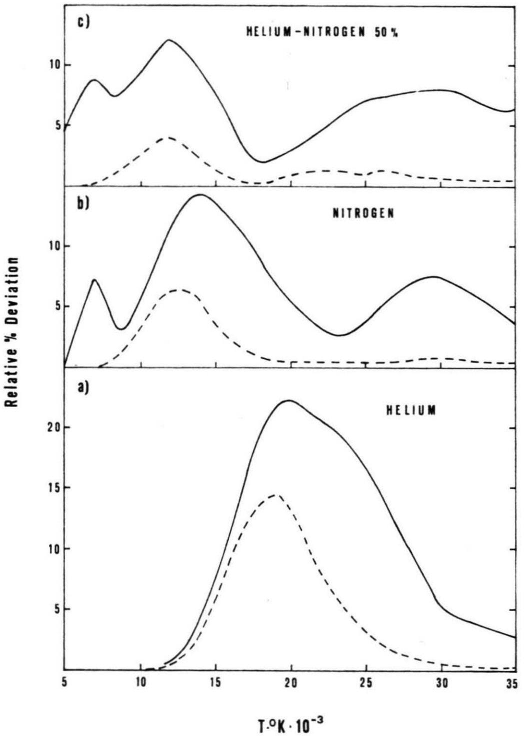

Fig. 1. ———— Values of $(\eta_1-\eta_f)/\eta_1\cdot 100$ as a function of temperature for different atmospheric plasmas. - - - - - - Values of $(\eta_1-\eta_s)/\eta_1\cdot 100$ as a function of temperature for different atmospheric plasmas.

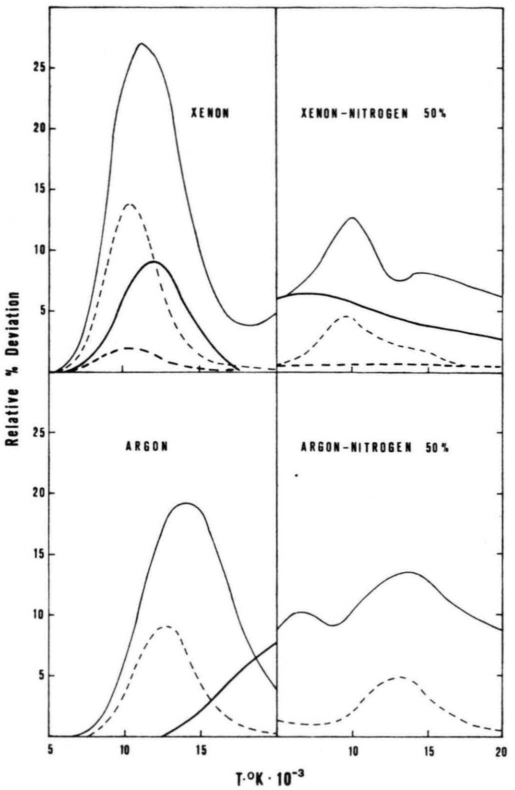

Fig. 2. ———— (thin lines) : Values of $(\eta_1-\eta_f)/\eta_1\cdot 100$ as a function of temperature for different atmospheric plasmas. - - - - - - - - - - - - (thin lines) : Values of $(\eta_1-\eta_s)/\eta_1\cdot 100$ as a function of temperature for different atmospheric plasmas. ———— (thick lines): Xe, Xe$-$N$_2$ 50%. Values of $(\eta_1-\eta_f)/\eta_1\cdot 100$ as a function of temperature calculated from $A^*_{\mathrm{Xe-Xe^+}}=A^*_{\mathrm{N-N^+}}=1$ — — — (thick lines): Xe, Xe$-$N$_2$ 50%. Values of $(\eta_1-\eta_s)/\eta_1\cdot 100$ as a function of temperature calculated from $A^*_{\mathrm{Xe-Xe^+}}=A_{\mathrm{N-N^+}}=1$ ———— (thick lines) : Ar . Values of $(\eta_2-\eta_1)/\eta_2\cdot 100$ (see text) for an atmospheric argon plasma.

In Figs. 1 and 2 the relative % deviation of Eqs. (4) and (5) with respect to Eq. (2) have been plotted as a function of the temperature for different plasmas. It can be seen that the convergence of Eqs. (4) and (5) for mixed plasmas is generally better than for the pure ones.

In order to explain this behaviour, let us consider the range of the first ionization, where the largest deviations are observed.

The main reason of the slow convergences of Eqs. (4) and (5) with respect to Eq. (2) for both pure and mixed plasmas can be ascribed to the fact that the off diagonal elements H_{ij}, when the couple ij is represented by the resonant ion-neutral interaction $\mathrm{M}-\mathrm{M}^+$, can not be neglected as compared with diagonal elements H_{ii}. This in turn depends on the fact that the A^*_{ij} factors appearing in the Eq. (2 b) are unusually small for $\mathrm{M}-\mathrm{M}^+$ interactions (see Table 1), since the charge transfer mechanism [12] abnormally increases the collision integrals diffusion type (i. e. the $\Lambda^{*(1,1)}_{ij}$), without alterating the collision integrals viscosity type (i. e. the $\Lambda^{*(2,2)}_{ij}$). This phenomenon is commonly disregarded [13–15] in the non resonant interactions $\mathrm{M}-\mathrm{N}^+$, $\mathrm{N}-\mathrm{M}^+$ so that for mixed plasmas the A^*_{ij} factors corresponding to these interactions are close to unity and the "non resonant" H_{ij} elements are less important than the "resonant" H_{ij} elements in determining the convergence of the approximations.

This point can be made more clear by evaluating the convergence of Eqs. (4), (5) on the assumption of an unrealistic value $A^*_{ij}=1$ for the resonant ion-neutral interactions. Figures 2 a – b show the strongly increased convergence in this case.

As a consequence the first approximation [Eq. (4)] is sufficient for mixed plasmas in an extended range of compositions, deviations being of the order of 10%, while the second approximation [Eq. (5)] is necessary for pure plasmas.

It should however be pointed out that η_1 [Eq. (2)] does not give exact values of the viscosity as compared with those calculated according to the true second approximation [16] of the Chapman-Enskog method (η_2) [Eq. (21) of Ref. [11]]. Fortunately the differences $\eta_2 - \eta_1$ become important in a temperature range in which the differences $\eta_1 - \eta_f$ or $\eta_1 - \eta_s$ can be neglected, as can be appreciated in Fig. 2 c for an argon plasma [17].

This in turn depends on the fact that the slow convergence $\eta_1 \rightarrow \eta_2$ is due to the charged-charged interactions [19] rather than to the ion-neutral interactions which produce the differences $\eta_1 - \eta_f$ or $\eta_1 - \eta_s$.

b) Translational Thermal Conductivity

The contribution of the heavy components to the translational thermal conductivity can be calculated with a very good accuracy by means of the Muckenfuss-Curtiss equation [7, 20], which represents the second approximation of the Chapman-Enskog method $(\lambda_H)_2$. This equation is known to assume a form very similar to Eq. (2), so that the expansions (3) to (5) also hold for $(\lambda_H)_2$.

One can therefore define a first $(\lambda_H)_f$ and a second approximation $(\lambda_H)_s$ to $(\lambda_H)_2$ by means of Equations (4) and (5).

The convergence of these approximations follows quite closely the behaviour discussed for the viscosity as shown by Fig. 3 and can be understood on the same basis.

Figure 3 also shows that the strong contribution of the electrons to the translational thermal conductivity of a plasma [21] λ_E, significantly reduces both the magnitude of these differences and the temperature interval in which they occur.

Table 1. Averaged A^*_{ij} values for resonant and non resonant ion-neutral interactions.

	He−He⁺	N−N⁺	Ar−Ar⁺	Xe−Xe⁺	Non Resonant ($M-N^+$)
A^*_{ij}	0.15	0.32	0.30	0.19	0.87

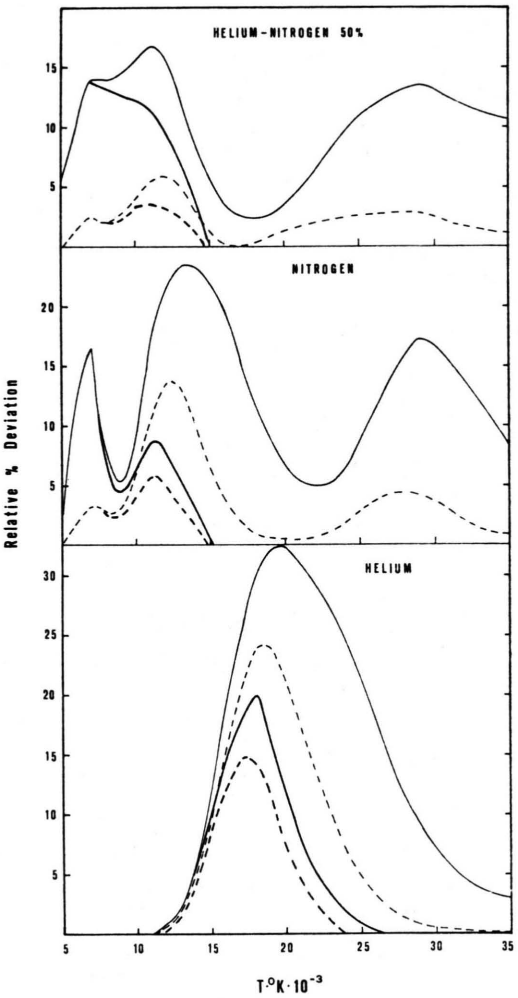

Fig. 3. ——— (thin lines): Values of $[(\lambda_H)_2-(\lambda_H)_f]/(\lambda_H)_2 \cdot 100$ as a function of temperature for different atmospheric plasmas. -------- (thin lines): Values of $[(\lambda_H)_2-(\lambda_H)_s]/(\lambda_H)_2 \cdot 100$ as a function of temperature for different atmospheric plasmas. ———— (thick lines): Values of $\{[(\lambda_H)_2+\lambda_E]-[(\lambda_H)_f+\lambda_E]\}/|(\lambda_H)_2+\lambda_E| \cdot 100$ as a function of temperature for different atmospheric plasmas. -------- (thick lines): Values of $\{[(\lambda_H)_2+\lambda_E]-[(\lambda_H)_s+\lambda_E]\}/[(\lambda_H)_2+\lambda_E] \cdot 100$ as a function of temperature for different atmospheric plasmas.

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Relaxation Phenomena in the Biological Carbon Cycle under Conditions of Variable Atmospheric CO_2-Content

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Under normal conditions the carbon cycle is a stationary process. A varying CO_2 partial pressure would cause deviations from the steady state. This process is analysed using a differentiated flow scheme of the carbon through the biosphere. For this purpose the biomass on land is divided into three reservoirs with average fixation times of 1 day (day-night-cycle), 2 years (annual period), and 100 years (timber, humus), respectively. The kinetic analysis shows that about 70% of the global respiration follow fluctuations of the rate of photosynthesis immediately, so that this portion of the excess of fixed carbon will be returned in the atmosphere within a few years. Only 30% of the total cycle passes through the largest reservoir (timber, humus) which could serve as a buffering system. But since the time constant of this partial cycle is about 100 years, the CO_2 exchange with the ocean is the faster process.

1. The Problem

In the natural carbon cycle CO_2, water, O_2, and organic material are cyclically transformed, and thus the stoichiometric relationships follow from the gross-equation (1):

$$CO_2 + H_2O \underset{\text{Respiration}}{\overset{\text{Photosynthesis}}{\rightleftarrows}} [CH_2O] + O_2 . \qquad (1)$$

With respect to the carbon, there is an almost completely cyclical conversion. From a geochemical material balance, which covers long periods of time, comes the result that a carbon atom will pass through cycle (1) 10,000 times on the average, before it is buried unoxidized in sediments, and in this manner is removed from the cycle [1]. The global conversion rates of photosynthesis and of respirative processes are thus equal to each other to a very good approximation. (In comparison, volcanism and carbonate formation have no importance.)

The material conversion rate on the land is estimated to be 10^{11} tons CO_2 per year [2]. This value depends on a number of factors which, however, from a global view, will change very slowly in comparison to the conversion time of carbon in the biological cycle; i. e., the average fixation time of car-

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